SRM VALLIAMMAI ENGINEERING COLLEGE  
B.E DEGREE EXAMINATION - MODEL EXAMINATION  
Fourth Semester  
Department of Mathematics  
MA8402 PROBABILITY AND QUEUEING THEORY  
(Regulations 2017)  
Time Three Hours  
Maximum 100 Marks

Answer ALL questions.

PART-A (10 X 2 = 20 marks)

1. Check whether the function given by \( f(x) = \frac{x+2}{25} \), for \( x = 1, 2, 3, 4, 5 \) can serve as the probability density function of a discrete random variable.

2. If a random variable \( X \) takes values 1, 2, 3, 4 such that \( 2P(X = 1) = 3P(X = 2) = P(X = 3) = 5P(X = 4) \). Give the probability distribution of \( X \).

3. Let \( X \) and \( Y \) have the joint probability mass function

\[
\begin{array}{c|ccc}
Y & 0 & 1 & 2 \\
\hline
0 & 0.1 & 0.4 & 0.1 \\
1 & 0.2 & 0.2 & 0
\end{array}
\]

Find \( P(X + Y > 1) \) and \( E(XY) \)

4. If \( X \) and \( Y \) have joint pdf \( f(x,y) = \begin{cases} x+y, & 0 < x < 1, 0 < y < 1 \\ 0, & \text{otherwise} \end{cases} \) Check whether \( X \) and \( Y \) are independent or not?

5. Give the properties of Poisson process.

6. Find the mean and variance of a stationary random process whose auto correlation function is given by

\[ R_{XX}(\tau) = 18 + \frac{2}{6 + \tau^2} \]

7. State the characteristics of a queuing system

8. If there are 2 servers in an infinite capacity Poisson queue system with \( \lambda = 10 \) per hour and \( \mu = 15 \) per hour, Examine the percentage of idle time for each server?

9. State the arrival theorem in the study of Jackson network

10. For an M/G/1 model if \( \lambda = 5 \) and \( \mu = 6 \) min and \( \sigma = 1/20 \), find the length of the queue.

PART-B (5 X 16 = 80 marks)

11. (a)i) A radar system has a probability of 0.1 of detecting a certain target during a single scan. Use Binomial distribution to find the probability that the target will be detected at least two times in four consecutive scans. Also compute the probability that the target will be detected at least once in 20 scans.

ii) A electrical firm manufactures light bulbs that have a length of which is normally distributed with mean \( \mu = 800 \) hours and standard deviation \( \sigma = 40 \) hours. Find the probability that a bulb burns between 778 and 834 hours.

(OR)

(b)i) A bolt is manufactured by 3 machines \( A, B, \) and \( C. \) \( A \) turns out twice as many items as \( B \) and machines \( B \) and \( C \) produce equal number of items. 2% of bolts produced by \( A \) and \( B \) are defective and 4% of bolts produced by \( C \) are defective. All bolts are put into 1 stock pile and 1 is chosen from this pile. What is the probability that it is defective?
ii) The probability of an individual suffering a bad reaction from an infection of a certain antibiotic is 0.001. Out of 2000 individuals, use Poisson distribution to find the probability that exactly three suffer. Also find the probability of more than 2 suffer from bad reaction.

12. (a)i) 20 dice are thrown. Find the approximate probability that the sum obtained is between 65 and 75 using central limit theorem

(ii) Two random variables X and Y have the following joint probability density function
\[ f(x, y) = \begin{cases} x + y; & 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0, & \text{otherwise} \end{cases} \]
Formulate the probability density function of the random variable \( U = XY \)

(b)ii) Two random variables X and Y have the joint density
\[ f(x,y) = \begin{cases} 2-x-y, & 0 < x < 1, 0 < y < 1 \\ 0, & \text{otherwise} \end{cases} \]
Calculate the Correlation coefficient between X and Y is \(-1/11\)

13. (a) Show that the random process \( X(t) = \cos \omega t + B \sin \omega t \) is wide sense stationary process if A and B are random variables such that \( E(A) = E(B) = 0 \), \( E(A^2) \) and \( E(AB) = 0 \)

i) A machine goes out of order whenever a component part fails the failure of this part is in accordance with a Poisson process with a mean rate of 1 per week. Find the probability that two weeks have lapsed since the last failure. If there are 5 spare parts of this component in inventory and the next supply is not due in ten weeks, find the probability that the machine will not be out of order in the next ten weeks.

(b)i) Let \( \{X_n : n = 1,2,3 \ldots \} \) be a Markov chain on the space \( S = \{1,2,3\} \) with one step t.p.m
\[
P = \begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 1 & 0 & 0 \end{bmatrix}
\]
1. Sketch the transition diagram, 2. Is the chain irreducible? Explain. 3. Is the chain ergodic? Explain. 4. Find the stationary distribution

ii) i) Prove that a Poisson Process is a Markov chain.
(ii) Prove that the difference of two independent Poisson process is not a Poisson process.
(iii) Prove that the sum of two independent Poisson process is a Poisson process. (iv) Find the mean and autocorrelation of the Poisson processes.

14. (a)i) Derive steady state probabilities of the number of customers in M/M/1 queuing system from the birth and death process and hence deduce that the average measures such as expected system size \( L_s \), queue size \( L_q \) expected waiting time in system \( W_s \), queue.

ii) There are three typists in an office. Each typist can type an average of 6 Letters per hour. If letters arrive for being typed at the rate of 15 letters per hour,
a) What fraction of the time all the typists will be busy? 
b) What is the average number of letters waiting to be typed?

c) What is the average time a letter has to spend for waiting and for being typed? 
d) What is the probability that a letter will take longer than 20 min waiting to be typed?
(b)i) A 2–person barber shop has 5 chair to accommodate waiting customers. Potential customers, who arrive when all 5 chairs are full, leave without entering barber shop. Customers arrive at the average rate of 4 per hour and spend an average of 12 min in the barber’s chair. Compute $P_0, P_1, P_7, E(N_q)$ and $E(w)$.

(8)

ii) On average 96 patients per (24 hour) day require the service of an emergency clinic. Also an average patient requires 10 minutes of active attention. Assume that the facility can handle only one emergency at a time. Suppose that it costs the clinic Rs. 100 per patient treated to obtain an average service time of 10 minutes, and that each minute of decrease in this average time would cost Rs .10 per patient treated. How much would have to be budgeted by the clinic to decrease the average size of the queue from $1 \frac{1}{3}$ patients to $\frac{1}{2}$ patient?

(8)

15. (a)i) Derive Pollaczek-Khinchin formula for M/G/1 queue. Hence deduce the result for M/D/1 and M/E_k/1 as a special case.

(8)

ii) In a departmental store, there are two sections namely grocery section and perishable section. Customers from outside arrive the G-section according to a Poisson process at a mean rate of 10 per hour and they reach the P-section at a mean rate of 2 per hour. The service times at both the sections are exponentially distributed with parameters 15 and 12 respectively. On finishing the job in G-section, a customer is equally likely to go to the P-section or leave the store, whereas a customer on finishing his job in the P-section will go to the G-section with probability 0.25 and leave the store otherwise. Assuming that there is only one salesman in each section, find (i) the probability that there are 3 customers in the G-section and 2 customers in the P-section (ii) the average waiting time of a customer in the store.

(8)

(b)i) Find $L_s, L_q, W_s$ and $W_q$. Automatic car wash facility operates with only one Bay. Cars arrive according to a Poisson process, with mean of 4 cars per hour and may wait in the facility’s parking lot if the bay is busy. (i) If the service time for all cars is constant and equal to 10 min (ii) Uniform distribution between 8 and 12 minutes (iii) Normal distribution with mean 12 minutes and SD 3 minutes (iv) Follows discrete distribution 4, 8 & 15 minutes with corresponding probability 0.2, 0.6 & 0.2

(8)

ii) In a two station service facility, queues are not allowed. Customers arrive at the facility at an average rate of 4 per hour; the server at each station serves at the rate of 5 customers per hour. If arrivals are Poisson and service times are exponential, find the probability that an arriving customers enter the system

(a) effective arrival rate

(b) Average (expected) number of customers in the system.

(c) Expected time of a customer spends in the system.

(8)