DEPARTMENT OF MATHEMATICS

QUESTION BANK

I SEMESTER
(COMMON TO ALL BRANCHES)

1918102 - ENGINEERING MATHEMATICS-I
Regulation – 2019
Academic Year 2019- 2020

Prepared by

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### QUESTION BANK

**SUBJECT**: 1918102 – Engineering Mathematics - I  
**YEAR /SEMESTER**: I Year / I Semester B.E./ B.Tech.  
(Common to all Branches)

**UNIT I  MATRICES**


<table>
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<th>Q.No</th>
<th>Question</th>
<th>BT-Level</th>
<th>Competence</th>
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<tbody>
<tr>
<td>1</td>
<td>Obtain the eigen values of $A^3$ where $A=\begin{pmatrix} 3 &amp; 2 \ 1 &amp; 2 \end{pmatrix}$</td>
<td>BTL-1</td>
<td>Remembering</td>
</tr>
<tr>
<td>2</td>
<td>Find the eigen values of $2A^2$ if $A=\begin{pmatrix} 4 &amp; 1 \ 3 &amp; 2 \end{pmatrix}$</td>
<td>BTL-1</td>
<td>Remembering</td>
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<tr>
<td>3</td>
<td>Find the sum and product of the eigen values of $A=\begin{pmatrix} 1 &amp; 2 &amp; -2 \ 1 &amp; 0 &amp; 3 \ -2 &amp; -1 &amp; -3 \end{pmatrix}$</td>
<td>BTL-1</td>
<td>Remembering</td>
</tr>
<tr>
<td>4</td>
<td>Find the sum and squares of the eigen values of $A=\begin{pmatrix} 3 &amp; 1 &amp; 4 \ 0 &amp; 2 &amp; 6 \ 0 &amp; 0 &amp; 5 \end{pmatrix}$</td>
<td>BTL-1</td>
<td>Remembering</td>
</tr>
<tr>
<td>5</td>
<td>The product of the 2 eigen values of $A=\begin{pmatrix} 6 &amp; -2 &amp; 2 \ -3 &amp; 3 &amp; -1 \ 2 &amp; -1 &amp; 3 \end{pmatrix}$ is 14. Find the 3rd eigen value.</td>
<td>BTL-1</td>
<td>Remembering</td>
</tr>
<tr>
<td>6</td>
<td>If the sum of 2 eigen values and the trace of a 3x3 matrix are equal , find the value of $</td>
<td>A</td>
<td>$</td>
</tr>
<tr>
<td>7</td>
<td>State Cayley-Hamilton theorem.</td>
<td>BTL-2</td>
<td>Understanding</td>
</tr>
<tr>
<td>8</td>
<td>Use Cayley Hamilton theorem to find $A^{-1}$ if $A=\begin{pmatrix} 2 &amp; 1 \ 1 &amp; -5 \end{pmatrix}$</td>
<td>BTL-2</td>
<td>Understanding</td>
</tr>
<tr>
<td>9</td>
<td>If $A=\begin{pmatrix} 1 &amp; 0 \ 4 &amp; 5 \end{pmatrix}$ find $A^3$ using Cayley Hamilton theorem</td>
<td>BTL-2</td>
<td>Understanding</td>
</tr>
<tr>
<td>10</td>
<td>Write any 2 applications of Cayley Hamilton theorem</td>
<td>BTL-2</td>
<td>Understanding</td>
</tr>
<tr>
<td>11</td>
<td>Prove that the eigen values of $A^{-1}$ are $\frac{1}{\lambda_1}, \frac{1}{\lambda_2}, \frac{1}{\lambda_3}, ....... \frac{1}{\lambda_n}$</td>
<td>BTL-3</td>
<td>Applying</td>
</tr>
<tr>
<td>12</td>
<td>Prove that sum of eigen values of a matrix is equal to its trace.</td>
<td>BTL-3</td>
<td>Applying</td>
</tr>
<tr>
<td>13</td>
<td>Find the sum of the eigen values of $2A$, if $A=\begin{pmatrix} 8 &amp; -6 &amp; 2 \ -6 &amp; 7 &amp; -4 \ 2 &amp; -4 &amp; 3 \end{pmatrix}$</td>
<td>BTL-3</td>
<td>Applying</td>
</tr>
<tr>
<td>14</td>
<td>Find the constants a and b such that the matrix $\begin{pmatrix} a &amp; 4 \ 1 &amp; b \end{pmatrix}$ has 3,-2 be the eigen values of A</td>
<td>BTL-4</td>
<td>Analyzing</td>
</tr>
<tr>
<td>15</td>
<td>For what values of c, the eigen values of the matrix $\begin{pmatrix} 1 &amp; 2 \ c &amp; 4 \end{pmatrix}$ are real and unequal , real and equal, complex conjugates.</td>
<td>BTL-4</td>
<td>Analyzing</td>
</tr>
<tr>
<td>16</td>
<td>Find the matrix corresponding to the quadratic form $2xy+2yz+2zx$.</td>
<td>BTL-4</td>
<td>Analyzing</td>
</tr>
<tr>
<td>17</td>
<td>Find the quadratic form corresponding to the matrix $A=\begin{pmatrix} 10 &amp; -2 &amp; -5 \ -2 &amp; 2 &amp; 3 \ -5 &amp; 3 &amp; 6 \end{pmatrix}$</td>
<td>BTL-5</td>
<td>Evaluating</td>
</tr>
<tr>
<td>18</td>
<td>Find the matrix corresponding to the quadratic form $x^2+y^2+z^2$</td>
<td>BTL-5</td>
<td>Evaluating</td>
</tr>
</tbody>
</table>
19 If 2, -1, -3 are the eigenvalues of the matrix A, then find the eigenvalues of $A^2 - 2I$

<table>
<thead>
<tr>
<th>PART-B</th>
</tr>
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<tbody>
<tr>
<td><strong>1(a)</strong> Find the eigenvalues and eigenvectors of the matrix $A = \begin{pmatrix} 1 &amp; 1 &amp; 3 \ 1 &amp; 5 &amp; 1 \ 3 &amp; 1 &amp; 1 \end{pmatrix}$</td>
</tr>
<tr>
<td><strong>1(b)</strong> Verify Cayley-Hamilton theorem and hence find $A^{-1}$ of $A = \begin{pmatrix} 1 &amp; 2 &amp; -2 \ -1 &amp; 3 &amp; 0 \ 0 &amp; -2 &amp; 1 \end{pmatrix}$</td>
</tr>
<tr>
<td><strong>2(a)</strong> Find the eigenvalues and eigenvectors of the matrix $A = \begin{pmatrix} 2 &amp; -2 &amp; 2 \ 1 &amp; 1 &amp; 1 \ 1 &amp; 3 &amp; -1 \end{pmatrix}$</td>
</tr>
<tr>
<td><strong>2(b)</strong> Test for the consistency of the following system of equations and solve them, if consistent $3x + y + z = 8, -x + y - 2z = -5, x + y + z = 6, -2x + 2y - 3z = -7.$</td>
</tr>
<tr>
<td><strong>3(a)</strong> Find the eigenvalues and eigenvectors of the matrix $A = \begin{pmatrix} 6 &amp; -2 &amp; 2 \ -2 &amp; 3 &amp; -1 \ 2 &amp; -1 &amp; 3 \end{pmatrix}$</td>
</tr>
<tr>
<td><strong>3(b)</strong> Examine the consistency of the equations $x + y + z = 3, 2x - y + 3z = 4, 5x - y + 7z = 11.$</td>
</tr>
<tr>
<td><strong>4(a)</strong> Obtain the eigenvalues and eigenvectors of the matrix $A = \begin{pmatrix} 2 &amp; 0 &amp; 1 \ 0 &amp; 2 &amp; 0 \ 1 &amp; 0 &amp; 2 \end{pmatrix}$ and verify that the eigen vectors are orthogonal in pairs.</td>
</tr>
<tr>
<td><strong>4(b)</strong> If $\lambda_1, \lambda_2, ..., \lambda_n$ are the eigenvalues of a matrix A, then (i) If $k\lambda_1, k\lambda_2, ..., k\lambda_n$ are the eigenvalues of the matrix $kA$, where $k$ is a non-zero scalar. (ii) $\lambda_1^p, \lambda_2^p, ..., \lambda_n^p$ are the eigenvalues of the matrix $A^p$, where $P$ is a positive integer. (iii) $\frac{1}{\lambda_1}, \frac{1}{\lambda_2}, ..., \frac{1}{\lambda_n}$ are the eigenvalues of $A^{-1}$.</td>
</tr>
<tr>
<td><strong>5(a)</strong> Verify Cayley-Hamilton theorem for the matrix $A = \begin{pmatrix} 1 &amp; 3 &amp; 7 \ 4 &amp; 2 &amp; 3 \ 1 &amp; 2 &amp; 1 \end{pmatrix}$ and also find $A^{-1}$.</td>
</tr>
<tr>
<td><strong>5(b)</strong> Obtain the eigenvalues and eigenvectors of the matrix $A = \begin{pmatrix} 0 &amp; 1 &amp; 1 \ 1 &amp; 0 &amp; 1 \ 1 &amp; 1 &amp; 0 \end{pmatrix}$</td>
</tr>
<tr>
<td><strong>5(c)</strong> Verify that the matrix $A = \begin{pmatrix} 2 &amp; -1 &amp; 2 \ -1 &amp; 2 &amp; -1 \ 1 &amp; -1 &amp; 2 \end{pmatrix}$ satisfies the characteristic equation and hence find $A^4$.</td>
</tr>
<tr>
<td><strong>6(a)</strong> Test whether the following system of equation possess a non-trivial solution $x_1 + x_2 + 2x_3 + 3x_4 = 0, 3x_1 + 4x_2 + 7x_3 + 10x_4 = 0, 5x_1 + 7x_2 + 11x_3 + 17x_4 = 0, 6x_1 + 8x_2 + 13x_3 + 16x_4 = 0.$</td>
</tr>
</tbody>
</table>
| **6(b)** Use Cayley-Hamilton theorem to find $\text{adj}(A)$ where
7(b) Investigate for the value of \( \lambda, \mu \) the system of equations
\( x + y + z = 6, x + 2y + 3z = 10, x + 2y + \lambda z = \mu \) have
(i) Unique solution, (ii) Infinitely many solution, (iii) No solution

8(a) Use Cayley-Hamilton theorem to find the value of the matrix
given by \( A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I \) if
the matrix \( A = \begin{pmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{pmatrix} \)

8(b) Find the non-trivial solution of the equation \( x + 2y + z = 0, 5x + y - z = 0, x + 5y + 3z = 0 \) if it exists.

9(a) Find the Eigen values of \( A \) and hence find \( A^n \), where \( n \) is
positive integer given that \( A = \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix} \)

9(b) Investigate for the value of \( \lambda \) for which the system of equations
\( 3x + y - \lambda z = 0, 4x - 2y - 3z = 0, 2\lambda x + 4y + \lambda z = 0 \)
possess a non-trivial solution, for this value of \( \lambda \) find the solution.

10 Reduce the quadratic form \( 8x^2 + 7y^2 + 3z^2 - 12xy + 4xz - 3yz \) into canonical form by orthogonal reduction.

11 Reduce the quadratic form \( 6x_1^2 + 3x_2^2 + 3x_3^2 - 4x_1x_2 - 2x_2x_3 + 4x_3x_1 \) into canonical form by an orthogonal reduction.

12 Reduce the quadratic form \( 3x_1^2 + 5x_2^2 + 3x_3^2 - 2x_1x_2 - 2x_2x_3 + 2x_3x_1 \) into canonical form by an orthogonal reduction.

13 Reduce the quadratic form \( 2x_1^2 + x_2^2 + x_3^2 + 2x_1x_2 - 2x_1x_3 - 4x_2x_3 \) into canonical form by an orthogonal reduction.

14 Reduce the quadratic form \( 2x_1^2 + 5x_2^2 + 3x_3^2 + 4x_1x_2 \) into canonical form by an orthogonal reduction.

Part C

1. Diagonalise the matrix \( A = \begin{pmatrix} 2 & 1 & -1 \\ 1 & 1 & -2 \\ -1 & -2 & 1 \end{pmatrix} \) by means of an
orthogonal transformation.

2. The Eigen vectors of a 3X3 real symmetric matrix \( A \) corresponding to the eigen values 2,3,6 are
\( (1, 0, -1)^T, (1, 1, 1)^T, (1, -2, 1)^T \) respectively. Find
the matrix \( A \).

3. Determine the nature of the quadratic form \( 2xy - 2yz + 2xz \) by reducing it into canonical form by orthogonal transformation

4. Determine the nature of the quadratic form \( x_1^2 + 2x_2^2 + x_3^2 - 2x_1x_2 + 2x_2x_3 \) by reducing it into
canonical form by orthogonal transformation

UNIT II DIFFERENTIAL CALCULUS
Limit of a function - Continuity – Differentiability - Differentiation rules – Roll’s Theorem
and Mean Value Theorem – Taylor’s Series - Maxima and Minima of functions of one
variable.

Q.No
. Question

| PART – A | BT Level | Domain |
1. **Show that** \( \lim_{x \to 0} x^2 \sin \left( \frac{1}{x} \right) = 0 \).  

2. **Using Rolle’s theorem find the value of c for the function**  
\[ f(x) = \sqrt{1 - x^2}, -1 \leq x \leq 1 \]  

3. **Evaluate** \( \lim_{x \to \frac{\pi}{2}} \frac{1 + \cos 2x}{2(\pi - 2x)^2} \).  

4. **Find** \( \lim_{x \to 0} \frac{\tan x - x}{x^3} \).  

5. **Use the squeeze theorem to show that** \( \lim_{x \to 0} \frac{\sqrt{x^3 + x^2} \sin \frac{\pi}{x}}{x} = 0 \).  

6. **Verify Lagrange’s law for the function**  
\[ f(x) = \frac{1}{x}, [1, 2] \]  

7. **Using Rolle’s theorem find the value of c for the function**  
\[ f(x) = (x - a)(b - x), a \leq x \leq b, a \neq b \]  

8. **Verify Lagrange’s law for the function**  
\[ f(x) = x^3, [-2, 2] \]  

9. **Find the Taylor’s series expansion of the function**  
\[ f(x) = \sin x \]  
about the point \( x = \frac{\pi}{2} \).  

10. **Point out** \( \frac{dy}{dx} \), if \( y = \ln|\cos(ln x)| \).  

11. **Does the curve**  
\[ y = x^4 - 2x^2 + 2 \]  
**have any horizontal tangents? If so there?**  

12. **Predict the values of a and b so that the function**  
\[ f(x) = \begin{cases} 1 & \text{if } x \leq 3 \\ ax + b & \text{if } 3 < x < 5 \text{ is continuous at } x=3 \text{ and } x=5. \\ 7 & \text{if } x \geq 5 \end{cases} \]  

13. **If the function**  
\[ f(x) = \begin{cases} \frac{x^2 - 16}{x - 4} & \text{if } x \neq 4 \\ c & \text{if } x = 4 \end{cases} \]  
**is continuous, what is the value of c?**  

14. **Where the function**  
\[ f(x) = |x| \]  
**is differentiable?**  

15. **Estimate**  
\[ \frac{d}{dx} (\sin x)^{\cos x} \]  

16. **Calculate**  
\[ \frac{d}{dx} \left( \sqrt[3]{x} \right) \]  

17. **Compute**  
\[ \frac{d}{dx} \left( \sqrt[x]{x} \right) \]  

18. **Evaluate**  
\[ \frac{d}{dx} \left( \sin x \ln x \right) \]  

19. **Estimate**  
\[ y’ \]  
**if**  
\[ x^3 + y^3 = 6xy \]  

20. **Find the critical numbers of the function**  
\[ f(x) = 2x^3 - 3x^2 - 36x \]  

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**PART – B**

1. **Point out the domain where the function f is continuous Also find the number at which the function f is discontinuous when**  
\[ f(x) = \begin{cases} 1 + x^2 & \text{if } x \leq 0 \\ 2 - x & \text{if } 0 < x \leq 2 \\ (x - 2)^2 & \text{if } x > 2 \end{cases} \]  

2. **Find**  
\[ \frac{dy}{dx} \]  
**if**  
\[ y = x^2 e^{2x}(x^2 + 1)^4. \]  

2. **Discuss the curve**  
\[ y = x^4 - 4x^3 \]  
**with respect to cancavity, points of inflection and local maxima and minima.**  

2. **Find the Taylor’s series expansion of**  
\[ f(x) = \frac{1}{1+x} \]  
**about x=0.**
3. (a) For what value of the constant “c” is the function “f” continuous on \((−∞, ∞), f(x) = \begin{cases} cx^2 + 2x; x < 2 \\ x^3 - cx; x \geq 2 \end{cases}\)

3.(b) (i) Find \(y'\) for \(\cos(xy) = 1 + \sin y\).
(ii) Find the first derivative of \(f(x) = \cos^{-1}\left(\frac{b + a \cos x}{a + b \cos x}\right)\)

4. (a) Find the absolute maximum and minimum of \(f(x) = x - 2 \tan^{-1} x\) in \([0,4]\).

4.(b) A rectangular sheet of metal square portions removed at the corners and the sides are then turned up so as to form an open rectangular box. Find the depth of the box so as the volume of the box is maximum.

5. (a) Find \(y'\) if \(x = a(\cos \theta + \log \tan \frac{\theta}{2}), y = a \sin \theta\).

5.(b) Using Rolle’s theorem, find the value of “c” if \(f(x) = 4x^3 - 9x, x \in \left[\frac{-3}{2}, \frac{3}{2}\right]\).

6. (a) Verify Lagrange’s law for the following \(f(x) = 2x^2 - 4x - 3, x \in [1,4]\).

6.(b) Use second derivative test to examine the relative maxima for \(f(x) = x(12 - 2x)^2\)

7. (a) Find the tangent line to the equation \(x^3 + y^3 = 6xy\) at the point \((3,3)\) and at what point the tangent line is horizontal in the first quadrant.

7. (b) Verify Rolle’s theorem for the following \(f(x) = 2x^3 - 5x^2 - 4x + 3, x \in \left[\frac{1}{2}, 3\right]\).

8. (a) Verify Lagrange’s law for the following \(f(x) = 2x^3 + x^2 - x - 1, x \in [0,2]\).

8.(b) Find where the function \(f(x) = 3x^4 - 4x^3 - 12x^2 + 5\) is increasing and where it is decreasing. Also find the local maximum and local minimum of \(f(x)\).

9. (a) Verify mean value theorem for the following \(f(x) = x^3 - 5x^2 - 3x, x \in [1,3]\).

9.(b) Find the local maximum and minimum values of \(f(x) = \sqrt{x} - \frac{4}{\sqrt{x}}\) by using both the first and second derivative tests.

10.(a) Find \(\frac{dy}{dx}\), when \(y = \frac{a \cos x + b \sin x}{b \cos x - a \sin x}\)

10.(b) Verify Rolle’s theorem for the following function \(f(x) = \sin x, 0 \leq x \leq \pi\)

11.(a) Find the Taylor’s series expansion of \(f(x) = \tan^{-1} x\) about \(x = 0\).

11.(b) If \(f(x) = 2x^3 + 3x^2 - 36x\), find the intervals on which it is increasing or decreasing, local maximum and minimum values of \(f(x)\).

12.(a) Show that the function \(f(x) = 1 - \sqrt{1 - x^2}\) is continuous in the interval \([-1, 1]\).

12.(b) Find \(y''\) if \(x^4 + y^4 = 16\).

13.(a) Find the Taylor’s series expansion of \(f(x) = \log(1 + x)\) about \(x = 0\).
13.(b) Examine the local extreme of \( f(x) = x^4 + 2x^3 - 3x^2 - 4x + 4 \). Also discuss the concavity and find the inflection points.

14.(a) Find \( \frac{dy}{dx} \) for the following functions \( e^x + e^y = e^{x+y} \).

14.(b) Verify Rolle’s theorem for the following \( f(x) = x(x-1)(x-2), x \in [0,2] \).

**Part C**

1. Find the point on the parabola \( y^2 = 2x \) that is close to the point (1,4).

2. A sheet of paper for a poster is to be 18 ft\(^2\) in area. The margin at top and the bottom are to be of inches, and the margins at the sides 6 inches. What should be the dimensions of the sheet to maximize the printed area?

3.(a) Apply Rolle’s theorem to find points on curve \( y = -1 + \cos x \), where the tangents is parallel to X axis in \( 0 \leq x \leq 2\pi \).

3(b) At what points on the curve \( x^2 - y^2 = 2 \), the slopes of tangents are equal to 2.

4(a) A cylindrical hole 4mm in diameter and 12mm deep in a metal block is rebored to increase the diameter to 4.12mm. Estimate the amount of metal removed.

4.(b) Find the equation of tangent at a point \((a, b)\) to the curve \( xy = e^2 \).

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**UNIT-III FUNCTIONS OF SEVERAL VARIABLES**

Partial derivatives – Total derivative Jacobians and properties - Taylor’s series for functions of two variables – Maxima and minima of functions of two variables – Lagrange’s method of undetermined multipliers

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<tr>
<td><strong>PART - A</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.</td>
<td>If ( u = \frac{y}{z} + \frac{x}{y} ), then find ( \frac{x}{\partial x} + \frac{y}{\partial y} + \frac{z}{\partial z} ).</td>
<td>BTL -1</td>
<td>Remembering</td>
</tr>
<tr>
<td>2.</td>
<td>If ( u = f(x, y, z, x, z) ), then find ( \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} ).</td>
<td>BTL -1</td>
<td>Remembering</td>
</tr>
<tr>
<td>3.</td>
<td>If ( x^y + y^x = 1 ), then find ( \frac{dy}{dx} ).</td>
<td>BTL -1</td>
<td>Remembering</td>
</tr>
<tr>
<td>4.</td>
<td>Find the value of ( \frac{du}{dt} ), given ( u = \log(x + y + z) ) where ( x = e^{-t} ), ( y = \sin t ), ( z = \cos t ).</td>
<td>BTL -2</td>
<td>Understanding</td>
</tr>
<tr>
<td>5.</td>
<td>Find the value of ( \frac{du}{dt} ), given ( u = x^3 + y^2 ), ( x = at^2 ), ( y = 2at ).</td>
<td>BTL -1</td>
<td>Remembering</td>
</tr>
<tr>
<td>6.</td>
<td>If ( u = x^3 y^2 + x^2 y^3 ) where ( x = at^2 ) and ( y = 2at ), then find ( \frac{du}{dt} ).</td>
<td>BTL -3</td>
<td>Applying</td>
</tr>
<tr>
<td>7.</td>
<td>Find ( \frac{du}{dt} ) if ( u = \sin \left( \frac{x}{y} \right) ), where ( x = e^t ), ( y = t^2 ).</td>
<td>BTL -3</td>
<td>Applying</td>
</tr>
<tr>
<td>8.</td>
<td>Find ( \frac{du}{dt} ) if ( u = \frac{x}{y} ), where ( x = e^t ), ( y = \log t ).</td>
<td>BTL -2</td>
<td>Understanding</td>
</tr>
<tr>
<td>9.</td>
<td>Find ( \frac{\partial r}{\partial x} ), if ( x = r \cos \theta ) &amp; ( y = r \sin \theta ).</td>
<td>BTL -3</td>
<td>Applying</td>
</tr>
<tr>
<td>10.</td>
<td>Find the Jacobian ( \frac{\partial (u, v)}{\partial (r, \theta)} ), if ( x = r \cos \theta ) &amp; ( y = r \sin \theta ), ( u = 2xy ), ( v = x^2 - y^2 ).</td>
<td>BTL -4</td>
<td>Analyzing</td>
</tr>
</tbody>
</table>
without actual substitution.

11. If \( u = \frac{y^2}{2x} \) and \( v = \frac{x^2 + y^2}{2x} \), find \( \frac{\partial(u,v)}{\partial(x,y)} \) BTL -3 Applying

12. If \( x = u/v, y = u/v \), find \( \frac{\partial^2(x,y)}{\partial(u,v)} \) BTL -1 Remembering

13. If \( u = x^y \) show that \( \frac{\partial^2u}{\partial x \partial y} = \frac{\partial^2u}{\partial y \partial x} \) BTL -2 Understanding

14. If \( u = \frac{x+y}{1-xy} \) and \( v = \tan^{-1} x + \tan^{-1} y \), find \( \frac{\partial(u,v)}{\partial(x,y)} \) BTL -3 Applying

15. Find the Taylor series expansion of \( x^y \) near the point (1, 1) up to first term BTL -2 Understanding

16. Expand \( xy + 2x - 3y + 2 \) in powers of \( (x-1) \) and \( (y+2) \), using Taylor’s theorem up to first degree form BTL -3 Applying

17. Find the Stationary points of \( f(x,y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x \). BTL -4 Analyzing

18. Find the Stationary points of \( x^2 - xy + y^2 - 2x + y \). BTL -4 Analyzing

19. State the Sufficient condition for \( f(x,y) \) to be extremum at a point BTL -4 Analyzing

20. Find the minimum point of \( f(x,y) = x^2 + y^2 + 6x + 12 \). BTL -4 Analyzing

**PART – B**

1.(a) If \( u = \log (x^2 + y^2) + \tan^{-1} \left( \frac{y}{x} \right) \), prove that \( u_{xx} + u_{yy} = 0 \) BTL -2 Understanding

1.(b) If \( u = \frac{yz}{x}, v = \frac{zx}{y} \) and \( w = \frac{xy}{z} \), find \( \frac{\partial(u,v,w)}{\partial(x,y,z)} \). BTL -3 Analyzing

2. (a) If \( u = f(2x - 3y, 3y - 4z, 4z - 2x) \), then find \( \frac{1}{2} \frac{\partial u}{\partial x} + \frac{1}{3} \frac{\partial u}{\partial y} + \frac{1}{4} \frac{\partial u}{\partial z} \) BTL -2 Understanding

2.(b) Find the Jacobian of \( \frac{\partial(x,y,z)}{\partial(r,\theta,\phi)} \) of the transformation \( x = r\sin\theta\cos\phi, y = r\sin\theta\sin\phi, z = r\cos\theta \) BTL -2 Understanding

3.(a) If \( u = (x^2 + y^2 + z^2)^{-1/2} \), then find the \( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \) BTL -4 Analyzing

3.(b) If \( x + y + z = u, y + z = uv, z = uvw \), prove that \( \frac{\partial(x,y,z)}{\partial(u,v,w)} = u^2v \) BTL -2 Understanding

4. (a) If \( u = f(x,y) \) where \( x = r\cos\theta, y = r\sin\theta \)
prove that \( \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial u}{\partial y} \right)^2 = \left( \frac{\partial u}{\partial r} \right)^2 + \left( \frac{1}{r^2} \frac{\partial u}{\partial \theta} \right)^2 \) BTL -4 Applying

4.(b) Examine \( f(x,y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x \) for extreme values. BTL -3 Analyzing

5. (a) If \( z = f(x,y) \) where \( x = u^2 - v^2, y = 2uv \)
prove that \( \frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2} = 4(u^2 + v^2) \left( \frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} \right) \) BTL -4 Applying

5.(b) Find the extreme values of \( f(x,y) = x^3 + y^3 - 3x - 12y + 20 \). BTL -3 Analyzing

6. (a) If \( u = x + y + z, u^2v = y + z \) and \( u^3w = z \) Show that \( \frac{\partial(u,v,w)}{\partial(x,y,z)} = u \) BTL -3 Analyzing

6.(b) Find the shortest distance from origin to the hyperbola \( x^2 + 8xy + 7y^2 = 225 \) BTL -3 Analyzing
7. (a) If \( u = \log(x^3 + y^3 + z^3 - 3xyz) \), show that \( \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z} \) \( u = -\frac{9}{(x+y+z)^2} \)

BTL -3 Analyzing

7. (b) Find the maximum and minimum distances of the point (3,4,12) from the sphere \( x^2 + y^2 + z^2 = 1 \).

BTL -2 Understanding

8. (a) Expand \( e^x \log(1 + y) \) in powers of \( x \) & \( y \) up to terms of third degree terms using Taylor’s series

BTL -1 Remembering

8(b) Discuss the maxima and minima of \( f(x,y) = x^3y^2(1 - x - y) \).

BTL -5 Evaluating

9. (a) Expand \( \tan^2 \frac{y}{x} \) in the neighborhood of \((1, 1)\) as Taylor’s series

BTL -3 Applying

9.(b) Find the Maximum value of \( x^n y^n z^n \) when \( x + y + z = a \).

BTL -3 Applying

10.(a) Find the Taylors series expansion of \( e^x \sin y \) at the point \((-1, \frac{\pi}{4})\) up to the third degree terms

BTL -4 Applying

10.(b) Find the extreme value of \( x^2 + y^2 + z^2 \) subject to the condition \( x + y + z = 3a \).

BTL -2 Understanding

11.(a) Expand \( e^{xy} \) in powers of \((x - 1)\) and \((y - 1)\) upto third degree terms by Taylor’s series

BTL -4 Applying

11(b) Find the maximum and minimum value of \( f(x, y) = 3x^2 - y^2 + x^3 \)

BTL -2 Understanding

12.(a) Expand Taylor’s series of \( x^3 + y^3 + xy^2 \) in powers of \((x - 1)\) and \((y - 2)\) upto the third degree terms.

BTL -5 Evaluating

12.(b) Find the volume of the greatest rectangular parallelopiped that can be inscribed in the ellipsoid \( \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \).

BTL -3 Applying

13.(a) If \( u = xyz, v = x^2 + y^2 + z^2 \) and \( w = x + y + z \) then find \( \frac{\partial (x,y,z)}{\partial (u,v,w)} \)

BTL -3 Applying

13(b) Find the shortest and longest distances from the point \((1,2,-1)\) to the sphere \( x^2 + y^2 + z^2 = 24 \)

BTL -4 Analyzing

14.(a) Find the dimension of the rectangular box without a top of maximum capacity, whose surface area is 108 sq. cm.

BTL -3 Applying

14.(b) Expand \( x^2 y^2 + 2 x^2 y + 3 xy^2 \) in powers of \((x + 2)\) and \((y - 1)\) using Taylor’s series up to third degree terms.

BTL -4 Applying

**Part C**

1. A wire of length \( b \) is cut into two parts which are bent in the form of a square and circle respectively. Find the least value of the sum of the areas so found using Lagrange’s method of multipliers.

BTL -6 Creating

2. The temperature at any point \((x,y,z)\) in space is given by \( \tau = kxyz^2 \), where \( k \) is constant. Find the height temp on the surface of the sphere \( x^2 + y^2 + z^2 = a^2 \).

BTL -5 Evaluating

3. A rectangular box open at top is to have a volume 32cc. Find the dimensions of the box that requires the least for its construction.

BTL -4 Applying

4. If the equation \( 5x^2 + 6xy + 5y^2 = 8 \) represents an ellipse with centre at the origin, find the lengths of its major and minor axes.

BTL -3 Applying

**UNIT IV INTEGRAL CALCULUS**
Definite and Indefinite integrals - Substitution rule - Techniques of Integration - Integration by parts, Trigonometric integrals, Trigonometric substitutions, Integration by partial fraction, Improper integrals.

<table>
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<tr>
<th>Q.No</th>
<th>Question</th>
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<th>Domain</th>
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<tbody>
<tr>
<td>1.</td>
<td>Prove that the following integral by interpreting each in terms of areas $\int_a^b x , dx = \frac{b^2-a^2}{2}$</td>
<td>BTL -1</td>
<td>Remembering</td>
</tr>
<tr>
<td>2.</td>
<td>State fundamental theorem of calculus</td>
<td>BTL -1</td>
<td>Remembering</td>
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<tr>
<td>3.</td>
<td>Evaluate $\int_0^1 \sqrt{1-x^2} , dx$ in terms of areas.</td>
<td>BTL -5</td>
<td>Evaluating</td>
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<tr>
<td>4.</td>
<td>If $f$ is continuous and $\int_0^4 f(x) , dx = 10$, find $\int_0^2 f(2x) , dx$</td>
<td>BTL -5</td>
<td>Evaluating</td>
</tr>
<tr>
<td>5.</td>
<td>Evaluate the integral $\int_a^b x , dx$ by using Riemann sum method</td>
<td>BTL -5</td>
<td>Evaluating</td>
</tr>
<tr>
<td>6.</td>
<td>Calculate $\int \frac{x^3}{\sqrt{4+x^2}} , dx$</td>
<td>BTL -3</td>
<td>Applying</td>
</tr>
<tr>
<td>7.</td>
<td>Calculate $\int \sqrt{1+x^2} , dx$</td>
<td>BTL -3</td>
<td>Applying</td>
</tr>
<tr>
<td>8.</td>
<td>Find $\int \sqrt{2x + 1} , dx$</td>
<td>BTL -3</td>
<td>Applying</td>
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<tr>
<td>9.</td>
<td>Find $\int \frac{x}{\sqrt{1-4x^2}} , dx$</td>
<td>BTL -3</td>
<td>Applying</td>
</tr>
<tr>
<td>10.</td>
<td>Evaluate $\int_0^1 \tan^{-1} x , dx$</td>
<td>BTL -5</td>
<td>Evaluating</td>
</tr>
<tr>
<td>11.</td>
<td>Calculate $\int \frac{(\ln x)^2}{x} , dx$</td>
<td>BTL -3</td>
<td>Applying</td>
</tr>
<tr>
<td>12.</td>
<td>Calculate $\int (\log x)^2 , dx$</td>
<td>BTL -3</td>
<td>Applying</td>
</tr>
<tr>
<td>13.</td>
<td>Evaluate $\int_0^1 \frac{dx}{(1+x)^4}$</td>
<td>BTL -2</td>
<td>Understanding</td>
</tr>
<tr>
<td>14.</td>
<td>What is wrong with the equation $\int_{-1}^4 \frac{4}{x^3} , dx = \frac{[-2x^{-2}]}{1} \bigg</td>
<td>_{-1}^{2} = \frac{3}{2}$?</td>
<td>BTL -5</td>
</tr>
<tr>
<td>15.</td>
<td>Evaluate $\int_{-1}^{\infty} \frac{1}{\sqrt{x}} , dx$ and determine whether it is convergent or divergent.</td>
<td>BTL -5</td>
<td>Evaluating</td>
</tr>
<tr>
<td>16.</td>
<td>Evaluate $\int_0^1 e^{-x^2} , dx$</td>
<td>BTL -5</td>
<td>Evaluating</td>
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<tr>
<td>17.</td>
<td>Estimate $\int_1^3 \sqrt{x^2 + 3} , dx$</td>
<td>BTL -5</td>
<td>Evaluating</td>
</tr>
<tr>
<td>18.</td>
<td>Evaluate the improper integral $\int_{2}^{3} \frac{dx}{\sqrt{3-x}}$, if possible.</td>
<td>BTL -5</td>
<td>Evaluating</td>
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<tr>
<td>19.</td>
<td>Find $\int_{2}^{\infty} \frac{dx}{\sqrt{3-x}}$</td>
<td>BTL -3</td>
<td>Analyzing</td>
</tr>
<tr>
<td>20.</td>
<td>Prove that $\int_{1}^{\infty} \frac{1}{x} , dx$ is divergent.</td>
<td>BTL -1</td>
<td>Remembering</td>
</tr>
</tbody>
</table>

**PART - B**

1. (a) Evaluate $\int \frac{(\ln x)^2}{x^2} \, dx$  
   BTL -5  Evaluating
1. (b) Calculate $\int \frac{1}{\sqrt{a^2-x^2}} \, dx$, by using trigonometric substitution. Hence use it to evaluate $\int \frac{x}{\sqrt{3-2x-x^2}} \, dx$,  
   BTL -3  Applying
2. (a) For what values of $p$ is $\int_{1}^{\infty} \frac{1}{xp} \, dx$ convergent?  
   BTL -5  Evaluating
2. (b) Find $\int x^3 \sqrt{9-x^2} \, dx$ by trigonometric substitution.  
   BTL -3  Applying
3. (a) Evaluate $\int_{0}^{\pi/2} \frac{\sin x \cos x}{\cos^2 x+3\cos x+2} \, dx$.  
   BTL -5  Evaluating
3. (b) Using trigonometric substitution evaluate $\int (5+4x-x^2) \, dx$  
   BTL -3  Applying
4. (a) Evaluate $\int x \tan^{-1} x \, dx$  
   BTL -5  Evaluating
4. (b) Evaluate $\int e^{ax} \cos bx \, dx$ using integration by parts  
   BTL -3  Applying
5. (a) Calculate using partial fraction \( \int \frac{10}{(x-1)(x^2+9)} \, dx \)

5.(b) Evaluate \( e^{-ax} \sin bx \, dx \), \( a > 0 \) using integration by parts.

6. (a) Prove that \( \int \tan^n x \, dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x \, dx \) \((n \neq 1)\).

6.(b) Find \( \int \frac{\sec^2 x}{\tan^2 x + 3 \tan x + 2} \, dx \)

7. (a) Evaluate \( \int_\frac{\pi}{4}^\frac{\pi}{3} x \tan x \, dx \).

7.(b) Use the substitution \( t = \tan \frac{x}{2} \), to transform the integral as a rational function of \( t \) and then evaluate \( \int \frac{1}{1+\sin x - \cos x} \, dx \)

8. (a) Evaluate \( \int \frac{2x + 5}{\sqrt{x^2 - 2x + 10}} \, dx \).

8.(b) Calculate by partial fraction \( \int \frac{\sqrt{2}x^2 + 1}{{(x-3)(x-2)^2}} \, dx \)

9. (a) Evaluate \( \int \frac{x e^{2x}}{(1+2x)^2} \, dx \)

9.(b) Compute \( \int \frac{x^3 + x^2 + 2x + 1}{(x^2 + 1)(x^2 + 2)} \, dx \) partial fraction.

10. (a) Evaluate \( \int \sin^6 x \cos^3 x \, dx \).

10.(b) Estimate \( \int e^{\tan^{-1} x} \left( \frac{1+x+x^2}{1+x^2} \right) \, dx \) by using an appropriate substitution.

11.(a) Evaluate \( \int_0^{\pi/2} \sin^7 x \cos^5 x \, dx \)

11.(b) Evaluate \( \int \frac{x}{\sqrt{x^2 + x + 1}} \, dx \)

12.(a) Prove the reduction formula \( \int_0^{\pi/2} \sin^n x \, dx = \frac{n-1}{n} \int_0^{\pi/2} \sin^{n-2} x \, dx \) . Hence by using it evaluate \( \int_0^{\pi/2} \sin^7 x \, dx \) and \( \int_0^{\pi/2} \sin^8 x \, dx \)

12.(b) Evaluate \( \int_0^\pi \cos^5 x \, dx \)

13.(a) Prove the reduction formula \( \int \cos^n x \, dx = \frac{1}{n} \cos^{n-1} x \sin x + \frac{n-1}{n} \int_0^{\pi/2} \cos x \, dx \) and use it to evaluate \( \int \cos^2 x \, dx \), \( \int \cos^4 x \, dx \), \( \int_0^{\pi/2} \cos^2 x \, dx \) and \( \int_0^{\pi/2} \cos^4 x \, dx \)

13.(b) Evaluate \( \int \frac{\tan x}{\sec x + \cos x} \, dx \)

14.(a) Prove that \( \int \sec^n x \, dx = \frac{\tan x \sec^{n-2} x}{n-1} - \frac{n-2}{n-1} \int \sec^{n-1} x \, dx \) \((n \neq 1)\)

14.(b) Evaluate the integral (i) \( \int_0^3 \frac{dx}{x^2 + 6x + 5} \) (ii) Show that \( \int_0^\infty e^{-x^2} \, dx \) is convergent.

**Part C**

1. Let \( A \) be the area of the region that lies under the graph of \( f(x) = x^2 \) between \( x=0 \) and \( x=1 \)

   (i) Using right end points

   (ii) Estimate the area by taking the sample points to be midpoints
and using four subintervals and then ten subintervals.

2. Let A be the area of the region that lies under the graph of \( f(x) = e^{-x} \) between \( x=0 \) and \( x=2 \)
   Estimate the area by taking the sample points to be midpoints and using (i) four subintervals and (ii) ten subintervals.

3. Let A be the area of the region that lies under the graph of \( f(x) = x^3 \) between \( x=0 \) and \( x=1 \)
   (i) Using right end points, find an expression for \( A \) as a limit
   (ii) Estimate the area by taking the sample points to be midpoints and using four subintervals.

4. Let A be the area of the region that lies under the graph of \( f(x)=1+x^2 \) between \( x=-1 \) and \( x=2 \)
   Estimate the area by taking the sample points to be midpoints
   (i) using three subintervals and (ii) using six subintervals.

UNIT V  MULTIPLE INTEGRALS
Double integrals in Cartesian and polar coordinates – Change of order of integration – Area enclosed by plane curves Change of variables in double integrals (Polar coordinates) – Triple integrals – Volume of solids

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<th>Q.No</th>
<th>Question</th>
<th>BT Level</th>
<th>Domain</th>
</tr>
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<tbody>
<tr>
<td>1.</td>
<td>Evaluate ( \int_{2}^{3} \int_{1}^{2} \frac{dxdy}{xy} )</td>
<td>BTL -5</td>
<td>Evaluating</td>
</tr>
<tr>
<td>2.</td>
<td>Evaluate ( \int_{0}^{\pi/2} \int_{0}^{\sin \theta} rdrd\theta )</td>
<td>BTL -2</td>
<td>Understanding</td>
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<tr>
<td>3.</td>
<td>Find the area bounded by the lines ( x = 0, y = 1 ) and ( y = x )</td>
<td>BTL -2</td>
<td>Understanding</td>
</tr>
<tr>
<td>4.</td>
<td>Evaluate ( \int_{0}^{\pi} \int_{0}^{a} rdrd\theta )</td>
<td>BTL -2</td>
<td>Understanding</td>
</tr>
<tr>
<td>5.</td>
<td>Evaluate ( \int_{0}^{\pi} \int_{0}^{a} r^2dxdy )</td>
<td>BTL -2</td>
<td>Understanding</td>
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<tr>
<td>6.</td>
<td>Evaluate ( \int_{0}^{a} \int_{\sqrt{a^2-x^2}}^{\sqrt{a^2}} dydx )</td>
<td>BTL -2</td>
<td>Understanding</td>
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<tr>
<td>7.</td>
<td>Evaluate ( \int_{1}^{\ln \theta} \int_{0}^{\ln y} e^{x+y}dxdy )</td>
<td>BTL -2</td>
<td>Understanding</td>
</tr>
<tr>
<td>8.</td>
<td>Evaluate ( \int_{0}^{\pi} \int_{0}^{a} r^4 \sin \theta drd\theta )</td>
<td>BTL -2</td>
<td>Understanding</td>
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<tr>
<td>9.</td>
<td>Evaluate ( \int_{0}^{\pi} \int_{0}^{x} \frac{dy}{x^2+y^2} )</td>
<td>BTL -2</td>
<td>Understanding</td>
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<tr>
<td>10.</td>
<td>Evaluate ( \int \int dxdy ) over the region bounded by ( x = 0, x = 2, y = 0 ) and ( y = 2 )</td>
<td>BTL -2</td>
<td>Understanding</td>
</tr>
<tr>
<td>11.</td>
<td>Change the order of integration ( \int_{0}^{1} \int_{y}^{x} f(x,y) dxdy )</td>
<td>BTL -2</td>
<td>Understanding</td>
</tr>
<tr>
<td>12.</td>
<td>Change the order of integration ( \int_{0}^{x} \int_{x}^{z} f(x,y) dxdy )</td>
<td>BTL -2</td>
<td>Understanding</td>
</tr>
<tr>
<td>13.</td>
<td>Find the limits of integration in the double integral ( \int_{R} \int f(x,y)dxdy ) where ( R ) is the first quadrant and bounded ( x=1, y=0, y^2 = 4x )</td>
<td>BTL -4</td>
<td>Applying</td>
</tr>
<tr>
<td>14.</td>
<td>Evaluate ( \int \int (x+y+z) dxdydz ) over the region bounded by ( x = 0, x = 1, y = 0 ) and ( y = 1, z = 0, z = 1 )</td>
<td>BTL -4</td>
<td>Applying</td>
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<td>15.</td>
<td>Write down the double integral to find the area of the circles ( r = 2\sin \theta, r = 4\sin \theta )</td>
<td>BTL -4</td>
<td>Applying</td>
</tr>
<tr>
<td>16.</td>
<td>Evaluate ( \int_{0}^{1} \int_{x}^{\sqrt{x}} xy(x+y)dydx )</td>
<td>BTL -3</td>
<td>Analyzing</td>
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</table>
17. Evaluate $\int_0^1 \int_0^{x^2} (x^2 + y^2) \, dy \, dx$

18. Evaluate $\int_1^3 \int_1^4 \int z \, dz \, dy \, dx$

19. Evaluate $\int_0^1 \int_0^{y^2} \int (x + y + z) \, dz \, dx$

20. Evaluate $\int_0^1 \int_0 \int_0^z e^x \, y + z \, dz \, dy \, dx$

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1. (a) Evaluate $\iint xy \, dxdy$ over the positive quadrant of the circle $x^2 + y^2 = a^2$

1. (b) Change the order of integration $\int_0^2 \int_0^{\sqrt{4-y^2}} xy \, dxdy$ and hence evaluate it

2. (a) Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} \sqrt{(a^2 - x^2 - y^2)} \, dxdy$.

2. (b) By change the order of integration and evaluate $\int_0^2 \int_{x^2}^{2-x} xy \, dy \, dx$

3. (a) Using double integral find the area of the Ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

3. (b) Change the order of integration $\int_0^{4a} \int_{\frac{x^2}{4a}}^{2\sqrt{ax}} xy \, dxdy$ and hence evaluate it

4. (a) By changing in to polar Co – ordinates , evaluate $\int_0^\infty \int_0^\infty e^{-\left(x^2+y^2\right)} \, dxdy$. Hence find the value of $\int_0^\infty e^{-x^2} \, dx$.

4. (b) Change the order of integration $\int_0^a \int_0^{2\sqrt{ax}} x^2 \, dxdy$ and hence evaluate it

5. (a) Evaluate $\int_0^2 \int_0^{\sqrt{2x-x^2}} (x^2 + y^2) \, dxdy$ by changing into polar co – ordinates

5. (b) Change the order of integration $\int_0^a \int_0^{\sqrt{ax}} x \, dy \, dx$ and hence evaluate it

6. (a) Express the volume of the sphere $x^2 + y^2 + z^2 = a^2$ as a volume integral and hence evaluate it.

6. (b) Find the area of the cardioids $r = a(1 + \cos \theta)$

7. (a) Find the area bounded by the parabolas $y^2 = 4 - x$ and $y^2 = x$

7. (b) Find the volume of the tetrahedrons bounded by the coordinate planes and $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$.

8. (a) Evaluate $\iint xy \, dxdy$ over the region bounded by the line $\frac{x}{a} + \frac{y}{b} = 1$ in the first quadrant.

8. (b) Evaluate $\iiint_V \frac{dxdydz}{(x+y+z+1)^3}$ where V is the region bounded by $x = 0, y = 0, z = 0$ and $x + y + z = 1$.

9. (a) Find the area included between the curves $y^2 = 4x$ and $x^2 = 4y$

9. (b) Evaluate $\int_1^e \int_{\log y}^x \int_1^x \log z \, dz \, dy \, dx$

10. (a) Change the integral into polar coordinates $\int_0^a \int_0^{x^2 \sqrt{x^2+y^2}} \, dy \, dx$

10. (b) Find the volume of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

11. (a) Find the area which is inside the circle $r = 3a \cos \theta$ and outside the cardioid $r = a (1 + \cos \theta)$. 

12. (b) Find the volume of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$
11.(b) Evaluate \[ \int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dxdydz}{\sqrt{1-x^2-y^2-z^2}} \] BTL -2 Understanding

12.(a) Find the area that lies inside the cardioid \( r = a (1 + \cos \theta) \) and outside the circle \( r = a \) by double integral BTL -3 Analyzing

12.(b) Find the value of \[ \iiint \text{xyz} \, dxdydz \] through the positive spherical octant for which \( x^2 + y^2 + z^2 \leq a^2 \). BTL -2 Understanding

13.(a) Evaluate \[ \iint_R \frac{xy}{\sqrt{x^2+y^2}} \, dxdy \] by converting into polar coordinates where \( R \) is the first quadrant part of the region bounded by two circles \( x^2 + y^2 = a^2 \) and \( x^2 + y^2 = 4a^2 \) BTL -2 Understanding

13.(b) Find the volume bounded by the cylinder \( x^2 + y^2 = 1 \) and the planes \( x + y + z = 3, z = 0 \) BTL -5 Evaluating

14.(a) Evaluate \[ \int_0^a \int_0^b \int_0^c (x^2 + y^2 + z^2) \, dxdydz \] BTL -6 Creating

14.(b) Find the area enclosed by the curves \( y^2 = 4ax \) and \( x^2 = 4ay \) BTL -2 Understanding

**Part C**

1. Find the area bounded by parabola \( y = x^2 \) and straight line \( 2x - y + 3 = 0 \). BTL -5 Evaluating

2. Find the area common to the cardioids \( r = a (1 + \cos \theta) \) and \( r = a (1 - \cos \theta) \) BTL -6 Creating

3. Find the volume of finite region of space (tetrahedron) bounded by the planes \( x = 0, y = 0, z = 0 \) and \( 2x + 3y + 4 = 12 \) BTL -5 Evaluating

4. Find the volume of sphere bounded by \( x^2 + y^2 + z^2 = a^2 \). BTL -6 Creating